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Interaction of Unsteady Expansion Wave with the Change of Cross-Sectional Area in Tube

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The interaction of an expansion wave with the cross-sectional area change in a shock tube was investigated both numerically and experimentally. Numerical results obtained using a piecewise-linear method (PLM) were discussed for the variation of the expansion wave characteristics and compared with the experimental results of a simple open-ended shock tube. Good agreement was obtained between calculated and measured pressure variations of the transmitted expansion wave. The relation between the maximum rate of pressure change and wave duration of transmitted wave was quantitatively clarified for the objective to design a negative impulsive wave generator.

Key Words: Compressible Flow, Expansion Wave, Shock Wave, Shock Tube, Impulsive Noise, High-Speed Railway Tunnel

1. Introduction

Pressure waves running in a tube are encountered very frequently in pneumatic transportation pipelines, pumps, nuclear power plants, and air brakes, as well as in rifles and high-speed railway tunnels. Problems on those pressure waves are, in general, difficult to solve because they are essentially unsteady and highly nonlinear. In the past works, the compression or shock waves running in a tube have been investigated experimentally and numerically, and distortion or attenuation of the pressure waves are now well documented. Researches on the centered-expansion waves and moving expansion waves have been mainly conducted in the shock tube facilities. Recently many researchers are concentrated on the moving expansion waves with regard to the wave phenomenon in high-speed railway tunnel, and they employ the expansion waves running in a straight tube that is a model tunnel (Kim, 1994).

The expansion waves are recently employed for obtaining a vacuum state, cooling or for controlling a prevailing state. The expansion wave running in a duct with or without its change of cross-sectional area can be handled by well-developed mathematical methods. In early works, the method of characteristics (MOC) was employed to predict nonstationary flows in ducts (Rudinger, 1955). The other numerical scheme was the random choice method (RCM) (Glimm, 1965; Chorin, 1976; Kashimura et al., 1986). The diffraction of expansion wave from a segment of an area change in a duct was investigated by Gottlieb & Igra (1983) and Igra & Gottlieb (1985) using the RCM. The purpose of their studies was to clarify the transient flow phenomenon that eventually establishes a quasi-steady flow, after all transient disturbances have subsided. They were not interested in the expansion wave itself. As will be described in the following sections, the present study is to employ the unsteady expansion wave for controlling the positive impulsive noise at the exit of high-speed railway tunnel.

Unsteady expansion wave running through a segment of cross-sectional area change in a duct was investigated numerically and experimentally. The objective of the present study is to obtain the maximum rate of pressure change and the time

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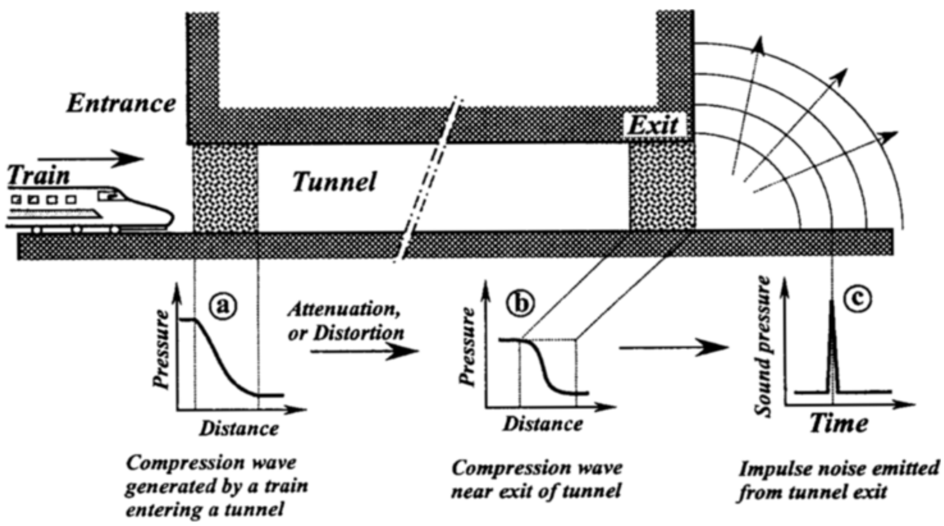


Fig. 1 Schematic diagram for impulse wave in high- speed railway tunnel

interval between the head and tail of the unsteady expansion wave. Thus the results of the present study are expected to be highly applicable to design the positive impulsive noise controller, i.e., the negative impulsive wave generator (NIWG).

2. Background of the Present Study

When a high-speed railway train enters a tunnel, as shown in Fig. 1, a compression wave is generated ahead of the train and propagates along the tunnel, compressing and accelerating the rest air in front of the wave. At the exit of the tunnel, an impulsive wave is emitted outward toward the surrounding, which causes a positive impulsive noise (Ozawa, 1979) like a kind of sonic boom produced by a supersonic aircraft. With the advent of high-speed train, such an impulsive noise emitted from a tunnel exit could cause the noise problem, unless some attempts are made to alleviate the pressure levels.

According to the aeroacoustic theory (Lighthill, 1952), the far-field sound pressure $\Delta p(r, t)$ resulting from a flow exhausted into atmosphere from the tunnel is given (Blake, 1986; Matsuo & Aoki, 1991);

$$\Delta p(r, t) \approx \frac{S_t}{\pi a_2 r}\left(\frac{\partial p}{\partial t}\right) \quad (1)$$

where r is the distance from the exit, t the time, S_t the cross-sectional area of a tunnel, a_2 the sonic velocity of atmosphere and p the pressure of compression wave respectively. Equation (1) shows that the maximum magnitude of an impulsive noise Δp_m caused by the emittion of compression wave at the tunnel exit is proportional to the maximum rate of pressure change of the compression wave front coming to the tunnel exit $(\partial p/\partial t)_m$.

In order to reduce the positive impulsive noise, the active noise cancellation (Stevens & Ahuja, 1991) which employs the additive source with an inverse phase, is more useful, although many passive controls such as perforated and flared tube extensions, slotted tunnel, and so on (Barrows, 1977; Vardy, 1978; Sockel, 1989) have been suggested so far. Here it should be noted that the passive controls are not very effective for the high-speed train/tunnel (Kim & Setoguchi, 1996a, b). For a reference, an actual arrangement of NIWGs is shown schematically in Fig. 2. The interaction between the positive impulsive wave discharged from a tunnel exit and the negative

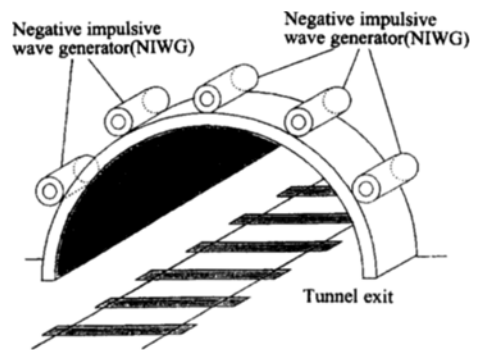


Fig. 2 Schematic arangement of negative impulsive wave generator

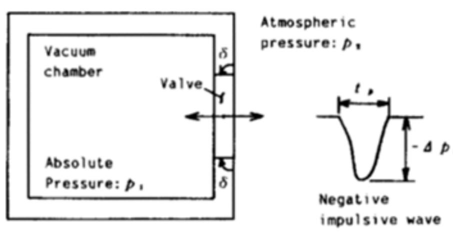


Fig. 3 Negative impulsive wave generator

impulsive wave generated from the generators reduces the sound pressure level of the tunnel impulsive noise. From the viewpoint of the methodology for noise cancellation, the NIWG may be effective for the tunnel impulsive noise. At present, there is, however, no equipment suitable to tunnel noise control and it is necessary to design the NIWGs. However the design of NIWG is difficult because of the lack of data on the unsteady expansion wave. For a design of NIWG, the characteristics of unsteady expansion wave should be fully understood. Figure 3 shows a NIWG that was first suggested by Setoguchi et al (1994).

The working principle of the generator is as follows. The chamber is first evacuated and as the valve installed on the chamber wall moves to the left (or right) side, the atmospheric air is induced into the chamber. Then the expansion wave is generated and emerged to the atmospheric side. The expansion wave creates a negative impulsive wave with its strength of $-\Delta p_m$ and its pulse

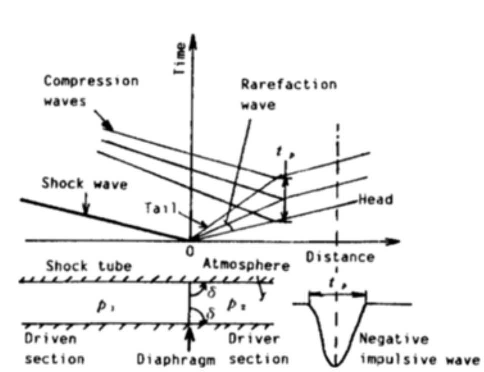


Fig. 4 Modeling of negative impulsive wave generator

duration of t_p. Here it should be noted that one of the most important design parameter of the generator is the valve angle δ, because the characteristics of expansion wave might highly depend on the passage geometry of the valve. For example, the passage is a convergent nozzle for $\delta < 90°$ and a divergent nozzle for $\delta > 90°$.

On the other hand, it is obvious from Eq. (1) that

$$-\Delta p_m \propto (-\partial p/\partial t)_m / r \qquad (2)$$

Equation (2) means that $-\Delta p_m$ is proportional to $(-\partial p/\partial t)_m$. In order to simulate the NIWG shown in Fig. 3, it is plausible to consider it a shock tube problem with an area change, as shown in Fig. 4. After a sudden removal of the diaphragm, a shock wave propagates into the driven gas and an expansion wave travels into the driver gas. The discharged expansion wave from the shock tube exit radiates toward the surrounding as the negative impulsive wave. Therefore it may be said that the shock tube itself is one of NIWGs. From a practical viewpoint of designing the NIWG, it should be noted that the performance of negative impulsive wave is determined by both $(-\partial p/\partial t)_m$ and t_p. The objective of the present study is to clarify the effect of the valve configuration on the $(-\partial p/\partial t)_m$ and t_p for the design of NIWG.

3. Numerical Calculation

The basic equations of continuity, momentum and energy for one-dimensional, non-heat conducting, unsteady, inviscid, compressible gas flows are

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\rho u}{S}\cdot\frac{dS}{dx}=0$$

$$\frac{\partial(\rho u)}{\partial t}+\frac{\partial(\rho u^2+p)}{\partial x}+\frac{\rho u^2}{S}\cdot\frac{dS}{dx}=0$$

$$\frac{\partial\left\{\rho\left(e+\frac{u^2}{2}\right)\right\}}{\partial t}+\frac{\partial\left\{\rho u\left(e+\frac{u^2}{2}\right)+up\right\}}{\partial x}+\frac{\rho u\left(e+\frac{u^2}{2}\right)+up}{A}\cdot\frac{dS}{dx}=0 \tag{3}$$

Here ρ is the density, u the velocity, p the pressure, e the internal energy per unit mass, t the time, x the axial coordinate and S the area of a duct respectively. In order to compute the interaction problem of an expansion wave with a cross-sectional area change in a shock tube, the variation of cross-sectional area $S(x)$ should be specified. As shown in Fig. 5, we assume that the cross-sectional area of the shock tube changes between two constant area ducts of the upstream area S_u and the downstream area $S_d(=D^2)$, respectively.

The segment of cross-sectional area change over a finite length l has a monotonic transition, as indicated in Eq.(4).

$$\frac{1}{S}\cdot\frac{dS}{dx}=\frac{2\tan\theta}{D+x\tan\theta} \tag{4}$$

The area change can be used for both enlargement and reduction cases and is chosen because it is advantageous over asymmetric variations in reducing numerical noises in the computed flowfield properties, where θ is the inclination angle of the area change.

Equation (3) was solved numerically using PLM (Leer, 1979; Colella & Glaz, 1983; Woodward & Colella, 1984; Saito et al., 1986), because this method was proven to be more suitable for solving such problems and superior to

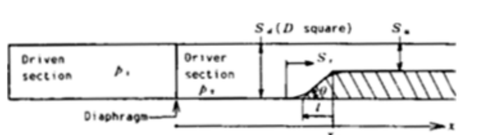


Fig. 5 Shock tube with cross-sectional area change

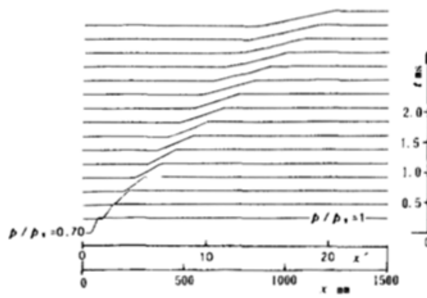
RCM that has been frequently used in the past. The operator splitting technique which was introduced to the RCM by Sod (1977) was adopted in the present study. The time increment Δt for computations was determined at each time step considering the CFL stability condition.

4. Experimental Facility

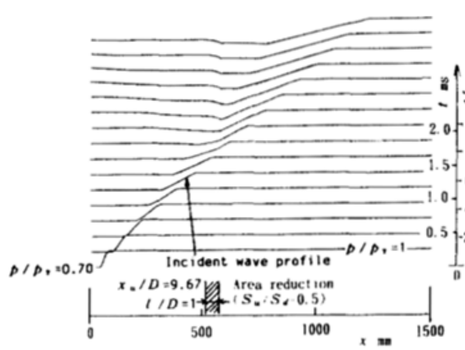
Static pressure measurements at some points in a shock tube with area changes are carried out to check the computed pressure histories for the expansion wave. The experimental apparatus used in the present study is the same as that of Fig. 5. The simple open-ended shock tube has a cross-sectional area of $S=60\times60\text{mm}^2$ with a total length of about 3.5m (the driven section length is about 2.2m). A calibrated pressure transducer (PCB:112A21) is mounted flash on the top wall of the shock tube. A sheet of cellophane of a thickness of 0.03mm was used as a diaphragm, and suddenly ruptured by a manually operated needle. Air at room temperature was used as working gas. The initial pressure ratio p_2/p_1 of the shock tube is set between 1.5 and 2.5, the driven air being initially at atmospheric pressure p_2. Output of the pressure transducer is recorded by an X-Y recorder by way of a wave memory. The uncertainty in pressure measurements is estimated to be ± 3 percent.

5. Numerical Results

The interaction of an expansion wave with a cross sectional area reduction ($S_u/S_d=0.5$) is shown in Fig. 6(b) in the form of spatial pressure distributions, together with the numerical results without an area change: see Fig. 6(a). Here, x, x', and x_u denote the distance measured from the diaphragm, the nondimensional distance of x/D and the position of area change end (S_u), respectively. t and t' are the time measured after instantaneous bursting of the diaphragm and the nondimensional time of $t/\{(D/a_2)\sqrt{\gamma}\}$, respectively, and the location of the segment of area change over the length l is also indicated by the hatching



(a) Without area change($S_u/S_d = 1.0$)



(b) With area change($S_u/S_d = 0.5$)

Fig. 6 Pressure distributions($p_2/p_1 = 2.0$, $a_1 = a_2 =$ 340m/s, $\gamma = 1.4$)

area. Each of the successive distributions is displaced upward from the previous one, such that a time-distance wave diagram is produced.

Figure 6(a) shows the rapidly spreading expansion wave and decreasing rates of pressure change with the travelling distance. This wave produces a flow toward the opposite direction to its propagating direction. Figure 6(b) shows that the subsequent interaction with the area reduction can be observed in the successive distributions, where the formation and evolution of the transmitted and reflected expansion waves can be seen, as well as the eventual development of steady subsonic flow in and on both sides of the area change. For a small area reduction, the reflected expansion wave is quite weak, relative to the incident wave, and barely noticeable. Note that the pressure in the area change decreases as the t or t′ increases. This is due to the increasing steady

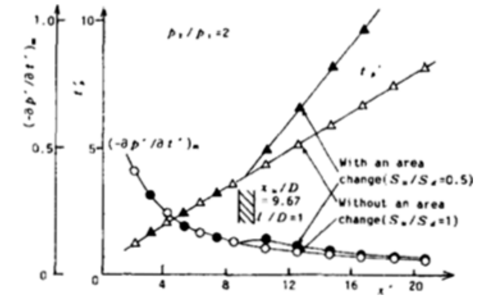


Fig. 7 Relation of expansion wave characteristics and distance measured from diaphragm

flow velocity as the gas moves from right to left through an area enlargement.

The variation in the expansion wave characteristics with nondimensional distance x′ is shown in Fig. 7, where one of the ordinates is the maximum rate of pressure change for expansion wave front $(\partial p'/\partial t')_m$ that corresponds to the value at the head of expansion wave. Here p' is the nondimensional pressure of p/p_2. The value of $(\partial p'/\partial t')_m$ decreases monotonically with increasing x'. This is easily understood because, for a centered expansion wave generated by instantaneous bursting of a diaphragm, the relation (Hall et al., 1974)

$$\left(-\frac{\partial p}{\partial t}\right)_{head} = a_2\left(-\frac{\partial p}{\partial x}\right)_{head} = \frac{2\gamma p_2}{(\gamma+1)t} = \frac{2\gamma p_2 a_2}{(\gamma+1)x} \qquad (5)$$

must be satisfied at the head, and gives the maximum pressure change (the initial rate of pressure variation). The value of $t_p' = [t_p/\{(D/a_2)\sqrt{\gamma}\}]$ increases linearly with the increase of x'. This is due to the fact that the time interval between head and tail of the centered expansion wave increases linearly with the distance measured from the diaphragm.

Comparison of the cases with and without an area change shows that the larger values of $(-\partial p'/\partial t')_m$ and t_p' can be obtained for the case with an area reduction. This is also true for all other numerical results for $S_u/S_d < 1$, as will be shown later. These are the advantageous features for the NIWG. Furthermore it is clear from Fig. 7 that the longer the length of driven section of the shock tube is, the weaker the strength of negative impulsive wave $(-\Delta p_m)$ is, and the wider the pulse duration (t_p) is. For the NIWG requiring the large strength in $(-\Delta p_m)$, the thin front-plate of vacuum chamber (see Fig. 3) is, therefore, recommended, while the thick front-plate is suitable for the purpose of requiring a wide pulse width.

Effect of length in the segment of area change l on the expansion wave characteristics is shown in Fig. 8, where x' is the location of pressure calculation. The value of $(-\partial p'/\partial t')_m$ is almost constant for all l/D_s. However t_p' decreases with increasing l/D for small l/Ds, and it approaches an asymptotic constant value. On the other hand, the expansion wave characteristics with the area change ratio can also be seen in Fig. 9, where $S_u/S_d<1$ and $S_u/S_d>1$ mean the cases of expansion waves passing through gradual area reduction and enlargement, respectively. Both values of $(-\partial p'/\partial t')_m$ and t_p' decrease with the increase in S_u/S_d. As mentioned in Fig. 7, it is obvious that the area reduction, i.e., the valve with the converging nozzle configuration, is favorable to the NIWG.

Figure 10 shows the effect of the position of the cross-sectional area change point x_u on the expansion wave characteristics, where the ordinates are t_p'/x' and $(-\partial p'/\partial t')_m x'$, respectively. For three x's, good correlations of t_p'/x' and $(-\partial p'/\partial t')_m x'$ are made against S_u/S_d. This tendency in $(-\partial p'/\partial t')_m x'$ is obvious from Eq. (5), which is the relation for a centered expansion wave. Also, the relation between t_p'/x' and x' is reasonable from the fact that t_p' is proportional to x for constant condition of p_2/p_1.

6. Comparison with Experimental Results

Figure 11 shows the comparison between calculated and measured pressure variations and the effect of initial pressure ratio for $S_u/S_d=1$. An examination of experimental pressure profile (dashed line) at the location of $x/D=11.8$ from the diaphragm shows that the expansion rate from experiment is markedly lower than the numerical one (broken line) at the same location. It is also found that the measured expansion rate at the head of the expansion wave is closer to the real flow than the theoretical one because there is always a finite opening time of the diaphragm (this means that the expansion wave is noncentered), while an instant burst of the diaphragm is

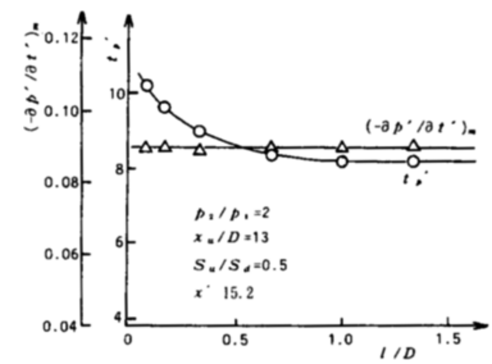


Fig. 8 Effect of length of the segment of area change on expansion wave characteristics

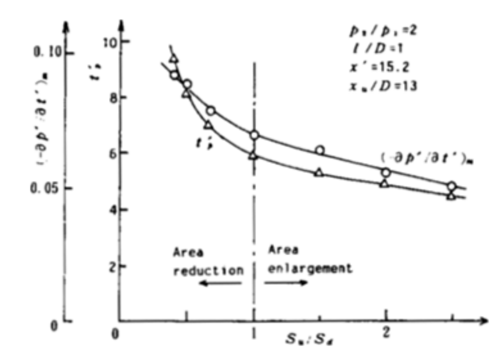


Fig. 9 Effect of area change on expansion wave characteristics

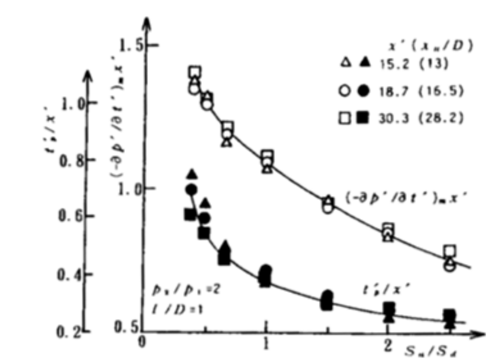


Fig. 10 Correlations of expansion wave characteristics

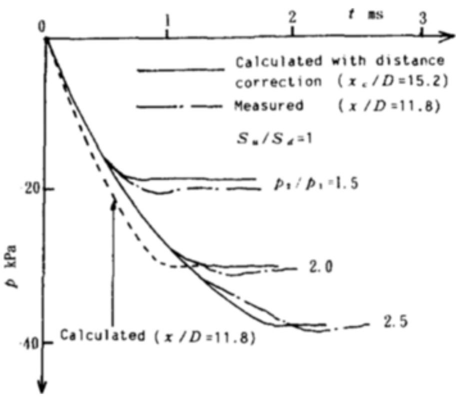


Fig. 11 Comparison of calculated and measured pressure variations

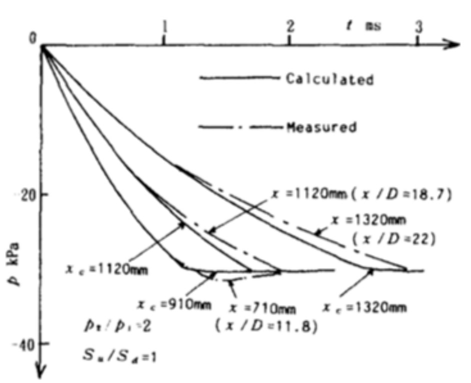


Fig. 12 Comparison of calculated and measured pressure variations

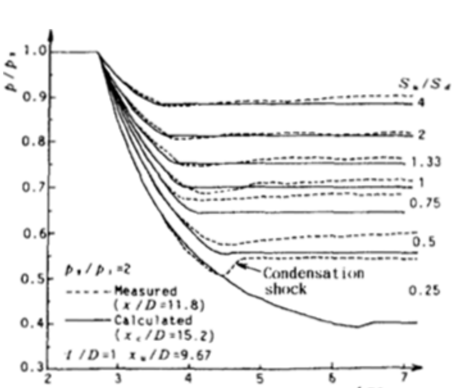


Fig. 13 Comparison of calculated and maasured pressure variations

well postulated for the theoretical analysis (Billington, 1956; Hall et al., 1974).

In order to apply the present noncentered waves to the centered waves, a correction can be made by using the measured speed of sound (Hall et al, 1974). The solid lines in Fig. 11 denote the numerical results taking the distance correction ($x_c/D=15.8$ for this case) into consideration. The agreement between numerical and experimental expansion behaviors is good although some deviations from the predicted profile appear near the tail of the expansion wave. It is clear from Fig. 11 that the magnitude of negative impulsive wave does not depend on p_2/p_1 because the value of $(-\partial p/\partial t)_{head}$ is the same for all the values of p_2/p_1. However, the pulse width depends on p_2/p_1 because a stronger expansion wave (i. e. for the larger p_2/p_1) has a wider fan of the characteristics. Figure 12 shows the effects of the distance measured from the diaphragm on the pressure histories. The calculated results agree well with the experimental ones. Here it should be noted that the expansion wave in a shock tube approaches a centered one when the length-to-diameter ratio is greater than about 15 (Courtney 1965). The present numerical results for $x/D=18.7$ do not take the distance correction into account.

Figure 13 shows the comparison between the calculated and measured pressure variations in terms of the area ratio. The calculated results agree very well with the measured pressure changes near the head of expansion wave for all the values of S_u/S_d. The values of t_p are also in good agreement with each other for $S_u/S_d>0.5$. But, for $S_u/S_d<0.75$, the experimental asymptotic pressures after the tail of expansion wave are not agreed with the calculated ones. This is because the condensation of atmospheric moist air occurs in the process of flow expansion (Courtney 1965). In particular, it can be observed from the figure that the condensation shock is generated in the expansion flow for $S_u/S_d=0.25$.

7. Conclusions

The interaction of expansion wave with a cross

-sectional area change in a shock tube is investigated both numerically and experimentally for the objective to design a negative impulsive wave generator. Numerical results obtained using piecewise-linear method were discussed for the variation of the expansion wave characteristics and compared with the experimental ones of a simple open-ended shock tube. The results showed good agreement between calculated and measured pressure variations of the transmitted expansion wave. The relation of both the maximum rate of pressure change and wave duration of transmitted wave was quantitatively clarified in terms of the cross-sectional area change of shock tube. This result suggests that the area reduction, which means the valve with the converging nozzle configuration, is favorable to the negative impulsive wave generator requiring the large strength and duration of the negative impulsive wave.

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